On Relay-Interference Networks with Quantized Feedback

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Abstract—We study quantized beamforming in wireless relay-interference networks with multiple transmitter-receiver pairs. For given transmitter rate requirements, we design structured distributed quantizers specifically to optimize the symbol error rate performance. We show that our quantizers achieve both maximal diversity and very high array gain using arbitrarily low feedback rates per receiver. Simulations are also provided, confirming our analytical results. We observe that our quantizers guarantee an equal diversity gain for each transmitter-receiver pair.

Index Terms—Wireless relay network, beamforming, interference, distributed vector quantization, symbol error probability, diversity gain, array gain.

I. INTRODUCTION

While it has been demonstrated in several studies that cooperation can greatly improve the performance and reliability of wireless network communications [1]–[4], interference still remains to be a fundamental issue in cooperative network design. Most of the previous work on cooperative networks relies on orthogonal channel allocation so that different transmitters do not interfere with each other. However, allocating orthogonal channels for each user may not be desirable due to time and bandwidth limitations. In such cases, one should explore effective ways to deal with interference while preserving cooperative diversity gains.

Multiple antenna interference cancelation techniques are very effective when dealing with interference in cooperative networks [5]. They offer reasonable performance with low decoding complexity. In this work, we consider a different approach. To be able to study the ultimate performance limits, we do not put any restrictions on our decoders. We would like to design a cooperation scheme that achieves maximal diversity benefits, and thus provides high reliability, even in the presence of multiuser interference.

For networks with a single transmitter-receiver pair and no interference, network beamforming using amplify-and-forward (AF) relays has shown to achieve the maximal spatial diversity [6], [7]. However, the optimal beamforming policy requires one or two real numbers to be broadcasted from the receiver to the relays. Using distributed beamforming with quantized instantaneous channel state information (CSI), it is possible to obtain both maximal diversity, as well as high array gain with only a few feedback bits from the receiver [8]–[10]. A special case of quantized feedback for cooperative networks is the relay selection scheme [11]–[13]. It has been formally shown in [8] that, for a network with \( R \) parallel relays, the relay selection scheme provides the maximum diversity \( R \).

Quantized feedback schemes have also been studied for other multiuser networks such as multiple-input multiple-output broadcast channels [14]. Some general discussions on quantizer design in networks can be found in [13].

This paper considers the quantized beamforming problem in interference networks with \( K \) transmitters, \( R \) parallel AF relays, and \( L \) receivers. Each transmitter and each relay has its own short term power constraint. We assume that the transmitters do not have any CSI. Each receiver knows its own receiving channels and the channels from the transmitters to the relays. Each relay only knows the magnitudes of its own receiving channels. Each relay and each receiver also has partial CSI provided by feedback.

Our performance measure is what we call the network error rate (NER). Given a fixed channel state, it is the probability that at least one user incorrectly decodes its desired symbol(s). In that sense, any receiver can be interested in the symbols transmitted by any subset of transmitters.

The rest of the paper is organized as follows: In Section II, we introduce our network model, performance and diversity measures, and problem definition. In Section III, we introduce our quantizer designs. Numerical results are provided in Section IV. Finally, in Section V, we draw our major conclusions.

Notation: \( \| \cdot \|_{\infty} \) is the infinite norm. \( \mathbb{C} \) and \( \mathbb{R} \) represent the sets of complex and real numbers, respectively. \( P \) represents the probability. \( E[X] \) is the expected value of \( X \). \( |\mathcal{A}| \) is the cardinality, and \( \mathcal{A}^n \) is the cartesian power of set \( \mathcal{A} \). \( \emptyset \) is the empty set. For a real-valued function \( f : \mathcal{C} \to \mathbb{R} \) with \( \mathcal{C} \subset \mathbb{C}^K \), let \( \mathcal{M} \triangleq \{ x : x \in \mathcal{C}, f(x) = \max_{x' \in \mathcal{C}} f(x') \} \). Then, \( \arg \max_{x \in \mathcal{C}} f(x) \) is the unique vector \( x^* \) with the property that \( x^* \prec x, \forall x \in \mathcal{M} \), and “\( \prec \)" represents some partial ordering (e.g. lexicographical ordering) of complex vectors. We define \( \arg \min(\cdot) \) in a similar manner. Finally, \( \log(\cdot) \) is the natural logarithm, and \( \log_2(\cdot) \) is the logarithm to base 2.

II. NETWORK MODEL AND PROBLEM STATEMENT

A. System Model

We have a relay network with \( K \) transmitters, \( L \) receivers, and \( R \) parallel relays, as shown in Fig. 1. The cases \( K = 1 \) and \( K > 1 \) correspond to a relay-broadcast network and a relay-interference network, respectively. We assume that there is no direct link between the transmitters and the receivers.

Denote the channel from the \( k \)th transmitter to the \( r \)th relay by \( f_{kr} \) and the channel from the \( r \)th relay to the \( l \)th receiver by \( g_{rl} \). Let \( \mathbf{h} = (f_{11}, \ldots, f_{KR}, g_{11}, \ldots, g_{RL}) \) denote the channel state of the entire network. We assume that the entries of \( \mathbf{h} \)
are independent and distributed as \( f_{kr} \sim \mathcal{CN}(0, \sigma^2_{f_{kr}}), g_{rt} \sim \mathcal{CN}(0, \sigma^2_{g_{rt}}) \) with finite variances \( \sigma_{f_{kr}}, \sigma_{g_{rt}} < \infty \), \( \forall r, k, \ell \). For brevity, let \( g_r \equiv (g_{1r}, \ldots, g_{Kr}) \), which denotes all the channels from the relays to the \( r \)th receiver.

Only the short-term power constraint is considered, which means that for every symbol transmission, the average power levels used at the \( k \)th transmitter and the \( r \)th relay are no larger than \( P_{Sk} \) and \( P_{Rr} \), respectively.

We assume a quasi-static channel model; the channel realizations vary independently from one channel state to another, while within each channel state the channels remain constant. We assume that the \( \ell \)th receiver knows \( g_r \) and each relay knows the magnitudes of its own receiving channels, i.e., the \( r \)th relay knows \( |f_{kr}|, k = 1, \ldots, K \). Each relay and each receiver also has partial CSI provided by feedback. In this paper, we consider two different feedback schemes, namely the global and local quantization schemes.

\[ x_n \]

\[ f \]

\[ g \]

\[ h \]

\[ \phi \]

\[ \psi \]

\[ \chi \]

\[ \Omega \]

\[ \Theta \]

\[ \Delta \]

\[ \Gamma \]

\[ \Lambda \]

\[ \Sigma \]

\[ \Xi \]

\[ \Psi \]

\[ \Omega \]

\[ \Theta \]

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\[ \Lambda \]

\[ \Sigma \]

\[ \Xi \]

\[ \Psi \]

\[ \Omega \]
The average power used at the \( r \)th relay can be calculated to be \( E_s(\ldots s_K r_p r_f [u_r]^2) = |x_r|^2 P_{R_r} s, \forall h. \) We require \( 0 \leq |x_r| \leq 1 \) as a result of the short term power constraint.

After the two steps of transmission, the received signal at the \( \ell \)th receiver can be expressed as \( y_{\ell} = \sum_{r=1}^{R} x_r \sqrt{P_r} g_{r \ell} r_f + \sum_{r=1}^{R} x_r \sqrt{P_r} g_{r \ell} h + \eta_{\ell}, \) where \( \eta_{\ell} \sim \mathcal{CN}(0,1) \) is the noise at the \( \ell \)th receiver.

We assume that the noises \( \eta_{\ell r}, r = 1, \ldots, R, \) and \( \eta_{\ell}, \ell = 1, \ldots, L \) are independent.

E. Performance Measures and Problem Statement

The \( \ell \)th receiver attempts to decode the symbols of the transmitters with indices given by an arbitrary but fixed set \( \mathcal{D}_\ell \subseteq \{1, \ldots, K\}, \mathcal{D}_\ell \neq \emptyset. \) Let us call the vector of transmitted symbols \( \mathbf{s}_r = [s_k]_{k \in \mathcal{D}_r} \) as the super-symbol relevant to the \( \ell \)th receiver, and \( \mathbf{s}_r \) be its decoded version. We say that an error event occurs at a receiver if it incorrectly decodes its desired super-symbol. In this case, the optimal decoder at the \( \ell \)th receiver is an individual maximum likelihood (ML) decoder given by \( \mathbf{s}_\ell = \arg \max_{s_k \in \mathcal{S}_k} P(s_k | y_r, x, h), \) where \( \mathcal{S}_\ell = \bigcap_{k \in \mathcal{D}_\ell} \mathcal{S}_k \) is the relevant super-symbol alphabet.

Let us now define a single quantity that represents the SER performance of all the receivers. We define the conditional network error rate (conditional NER, or CNER), denoted by CNER, as the probability that at least one receiver incorrectly decodes its desired super-symbol. A useful upper-bound on the CNER [15] is \( \text{CNER} \leq C_0 \exp(-\gamma^L(x, h)), \) where \( C_0 \triangleq L(|\mathcal{S}| - 1)/4, \mathcal{S} = \bigcap_{\ell=1}^{L} \mathcal{S}_\ell, \) and

\[
\gamma^L(x, h) = \min_{\ell} \min_{s \in \mathcal{S}_\ell} \min_{\tilde{s} \in \mathcal{S}} \frac{1}{4(1 + \sum_{r=1}^{R} P_r |g_{r \ell} x_r|^2)} \frac{\sum_{k=1}^{K} (s_k - \tilde{s}_k)^2 \sqrt{P_S}\sum_{r=1}^{R} \sum_{i=1}^{g_{r \ell}} f_i g_r x_r}{\sum_{r=1}^{R} P_r |g_{r \ell} x_r|^2}
\]

is what we call the network signal-to-noise ratio (NSNR) that characterizes the overall performance of the network.

Our performance measure, the NER, is the expected value of the CNER. Given a quantizer \( Q \) global or local, the NER can thus be expressed as \( \text{NER}(Q) \triangleq E_n \text{CNER}(Q(x), h) ). \)

Let us also define a unique diversity measure for our network. Let \( P_{R_r} = P_{R}, r = 1, \ldots, R, P_{S_k} = p_{S_k}, k = 1, \ldots, K, \) where \( p_{S_k} > 0. \) In other words, we allow the power constraint of each transmitting terminal to grow linearly with \( P. \) Then, the first-order diversity achieved by a quantizer \( Q \) is given by \( d_1(Q) \triangleq \lim_{P \to \infty} - \log \text{NER}(Q)/\log P. \)

One problem with this conventional definition of diversity is that it fails to characterize the asymptotic effect of possible sub-linear \( P \)-dependent terms (e.g. logarithmic terms) in the error rate expression. In order to properly handle such cases, we define the second-order diversity as \( d_2(Q) \triangleq \text{lim}_{P \to \infty} - \log \text{NER}(Q) + d_1(Q) \log P) / \log P. \)

Now, the diversity (gain) achieved by a quantizer \( Q \) is given by \( d(Q) \triangleq (d_1(Q), d_2(Q)), \) and the asymptotic performance of a quantizer \( Q, \) as \( P \to \infty, \) can be expressed as \( \text{NER}(Q) \approx G_k(P) / \log P)^{-d_2(Q)} P^{-d_1(Q)}. \) The factor \( G_k(P) \) is the array gain, and it satisfies \( \lim_{P \to \infty} G_k(P) / \log P = 0. \) For diversity gains \( d = (d_1, d_2), \) and \( d' = (d'_1, d'_2), \) we say that \( d \) is higher than \( d' \) (or \( d > d' \)) if either \( d_1 > d'_1 \) or \( d_2 > d'_2. \) The other inequalities are defined similarly.

It can be shown [15] that for any quantizer \( Q, d(Q) \leq (R, 0) \) if \( K = 1, \) and \( d(Q) \leq (R, -R) \) if \( K > 1. \) In other words, interference results in a second order diversity loss regardless of the type or the resolution of the quantizer that is used.

Fortunately, there exists a relay-selection based GQ, namely \( G_{Q_c}_2(h) \triangleq \arg \max_{e_r \in e_r} \gamma^L(e_r, h) \)

that can achieve maximal diversity for any \( K, R, \) and \( L [15]. \) In \( (1), G_{Q_c} \triangleq (e_1, \ldots, e_L) \) represents the relay selection codebook with \( e_{r q} = 1 \) for \( q = r, \) and \( e_{r q} = 0 \) for \( q \neq r. \) Hence, \( G_{Q_c} \)

III. DIVERSITY WITH LQS

We construct our LQs using the localization method [16], in which we synthesize an LQ out of an existing GQ. The synthesized LQ and the GQ share the same codebook. For our particular quantization scheme, we use the relay-selection based GQ in (1) as the basis of our LQs.

A. Localization

Let \( LQ_{\xi,N} \) denote a generic localization of \( G_{Q_c}. \) For the synthesized quantizer \( LQ_{\xi,N}^{(h)} \), the superscript indicates whether it is fixed-length (f) or variable-length(v); and \( \xi, N \) are design parameters that we shall specify later on. For a particular channel state \( h, \) the components of the synthesized quantizer operate as follows:

1) LQ Encoders: For notational convenience, \( \omega_{r \ell} = \gamma^L(e_r, h). \) The \( \ell \)th LQ encoder calculates \( \omega_{r \ell}, r = 1, \ldots, R. \) In other words, it calculates its own contribution to the NSNR for all possible relay selection vectors. Then, it quantizes each of the possible contributions using a scalar quantizer \( N(x) = \begin{cases} n, & \exists n \in \{0, \ldots, N-2\}, x \in [n\xi, (n+1)\xi), \\ N, & \text{otherwise}. \end{cases} \)

Its output message is the concatenation of \( R \) sub-messages \( N(\Omega_{r \ell}), r = 1, \ldots, R. \)

2) Compressors: There are \( R \) sub-messages, each with \( N \) possible values. Therefore, for a fixed-length synthesis \( LQ_{\xi,N}^{(h)}, \) at each channel state, each receiver feeds back \( [R \log_2 N] \) bits without any compression.

For a variable-length synthesis \( LQ_{\xi,N}^{(h)} \), we use a loss-less compressor that produces an empty codeword whenever \( N(\Omega_{r \ell}) = N, \ \forall r, \) and otherwise a codeword of length \( \lceil \log_2 \lceil \log_2 N \rceil - 1 \rceil \) bits that can uniquely represent each \( N(\Omega_{r \ell}). \) Hence, for a given channel state, the number of feedback bits produced by any receiver is either 0 or \( \lceil \log_2 \lceil \log_2 N \rceil - 1 \rceil. \)
After all the \( L \) feedback messages of the receivers are exchanged between the receivers and the relays, each of them decodes the feedback bits using the local decoder. The decoder comprises of a decompressor and an LQ decoder, and its operation is the same for each receiver and relay.

3) **Decompressor:** The decompressor recovers all the sub-messages from all the receivers, \( N'(\omega _\ell r), \ r = 1, \ldots , R, \ \ell = 1, \ldots , L \). These are then passed to the LQ decoder.

4) **LQ Decoder:** The main goal of the LQ decoder is to imitate the GQ as good as possible. For that purpose, let \( R_g \equiv \{ q : \min _l \omega _{\ell q} = \max _r \min _\ell \omega _{\ell r} \} \) denote the set of indices from which our GQ in (1) produces its output. In other words, \( R_g \) is the set of indices of relays that provide the maximal NSNR. Also, let \( R_l \equiv \{ q : N(\min _\ell \omega _{\ell q}) = N(\max _r \min _\ell \omega _{\ell r}) \} \). Note that \( R_g \subset R_l \). Moreover, due to the structure of \( N \), not only \( N(\min _\ell \omega _{\ell q}) = \min _\ell N(\omega _{\ell q}) \), but also \( N(\max _r \min _\ell \omega _{\ell r}) = \max _l N(\omega _{\ell r}) = \max _r \min _l N(\omega _{\ell r}) \). Therefore, \( R_l \) can be easily calculated by the LQ decoder.

Since \( R_g \subset R_l \), the LQ decoder can determine which relay selection vector(s) can possibly provide the maximal NSNR. In general, it can choose any one of the relay selection vectors that are indexed by \( R_l \). But, to be more precise, we define

\[
\text{LQ}_{\xi,N}^d(h) \triangleq \arg \max _{c_\ell \in C_R} \min _\ell N(\omega _{\ell r}).
\]

5) **Localization Distortion:** Let us now study two possible cases of interest regarding the LQ output: If \( R_g = R_l \), then the LQ output provides the same NSNR as the GQ output. Otherwise, the LQ might make a suboptimal decision. This results in what we call the localization distortion (LD), i.e.,

\[
\text{LD}(\xi, N) \triangleq \text{NER}(\text{LQ}_{\xi,N}^d) - \text{NER}(\text{GQ}_{\xi,N}^d).
\]

We can now design our LQs by appropriately choosing the scalar quantizer parameters \( \xi \) and \( N \). Moreover, we can also determine their performance analytically, by using (2).

**B. Maximal First-Order Diversity with an fLQ**

We first design an fLQ that uses a fixed \( R \) feedback bits per channel state per receiver. The following theorem shows that our fLQ achieves maximal first-order diversity.

**Theorem 1.** Let \( \xi = \log _2 R \) and \( N = 2 \). Then, for \( P \) sufficiently large, the NER with \( \text{LQ}_{\xi,N}^d \) is upper bounded by

\[
\begin{align*}
\text{NER}(\text{LQ}_{\xi,N}^d) &\leq C_5 \log _2^2 P, & K = 1, \\
\text{NER}(\text{LQ}_{\xi,N}^d) &\leq C_6 \log _2^2 P, & K > 1.
\end{align*}
\]

for constants \( 0 < C_5, C_6 < \infty \) that are independent of \( P \).

**Proof:** The proof is provided in [15].

In other words, using a fixed \( R \) feedback bits per receiver per channel state, we can achieve diversity \((R, -R)\) for \( K = 1 \), and diversity \((R, -2R)\) for \( K > 1 \). Since \((R, -R) < (R, 0)\) for the broadcast network, and \((R, -2R) < (R, -R)\) for the interference network, our fLQ has a second-order diversity loss compared to the optimal performance.

The scalar quantizer resolution for our fLQ is \( \log _2 N = 1 \) bit per local NSNR. In what follows, we show that, by appropriately increasing the resolution with \( P \), one can achieve maximal diversity, while the compressors make sure that the feedback rate remains bounded.

**C. Maximal Diversity with a vLQ**

For vLQs equipped with entropy coding, we have the following result:

**Theorem 2.** Let \( \epsilon > 0 \) be a fixed constant that is independent of \( P \). For any \( \Lambda \) with \( 0 < \epsilon < \Lambda \leq P \), let \( \xi = \frac{1}{\Lambda} \), and

\[
\begin{align*}
N_r &\equiv [\Lambda \log \Lambda + RA \log P + 1], & K = 1, \\
N_r &\equiv [\Lambda \log \Lambda + RA \log (\frac{P}{\log P}) + 1], & K > 1.
\end{align*}
\]

Then, for \( P \) sufficiently large, we have

\[
\begin{align*}
\text{LD}(\xi_r, N_r) &\leq C_7 \frac{\log P}{\Lambda}, & K = 1, \\
\text{LD}(\xi_r, N_r) &\leq C_8 \frac{\log B P}{\Lambda}, & K > 1,
\end{align*}
\]

and, in addition, the feedback rate of the \( r \)th receiver satisfies

\[
\begin{align*}
\text{R}_r(\text{LQ}_{\xi,N}^d) &\leq C_9 \frac{\log P}{\Lambda}, & K = 1, \\
\text{R}_r(\text{LQ}_{\xi,N}^d) &\leq C_{10} \frac{\log B P}{\Lambda}, & K > 1,
\end{align*}
\]

where \( 0 < C_7, C_8, C_9, C_{10} < \infty \) are constants that are independent of \( \Lambda \) and \( P \).

**Proof:** The proof is provided in [15].

We now describe several consequences of this theorem for \( K > 1 \). The consequences for \( K = 1 \) will be analogous.

Let us first note that since \( \text{GQ}_{\Lambda} \) achieves full diversity, we have \( \text{NER}(\text{GQ}_{\Lambda}) \leq C_4 \frac{\log P}{P} \) for some constant \( 0 < C_4 < \infty \). An upper bound on \( \text{LD}(\xi_r, N_r) \) is given by Theorem 2. Substituting the two bounds to (1), we have \( \text{NER}(\text{LQ}_{\xi,N}^d) \leq (C_4 + C_8 \Lambda^{-1}) \frac{\log P}{P} \). In other words, our vLQ achieves maximal diversity. But we also have \( \text{NER}(\text{LQ}_{\xi,N}^d) \leq \text{NER}(\text{GQ}_{\Lambda}) + C_{11} \frac{\log P}{P} \). Thus, by increasing \( \Lambda \), the array gain performance of our vLQ can be made arbitrarily close to the one provided by the GQ, at any power level \( P \).

What is more interesting is the behavior of the upper bound on the feedback rate given by (3). As \( P \to \infty \), the required feedback rate decays to zero. In other words, both the diversity and array gain benefits of \( \text{NER}(\text{GQ}_{\Lambda}) \) can be achieved using arbitrarily low feedback rates, when \( P \) is sufficiently large.

**IV. Simulation Results**

In this section, we present numerical evidence that verifies our analytical results. We assume that each receiver attempts to decode all the symbols from all the transmitters. In other words, \( D_r = \{ 1, \ldots , K \}, \forall r \). In the graphs, “GQ” represents \( \text{GQ}_{\Lambda} \) in (1), “fLQ” denotes \( \text{LQ}_{\xi,N}^d \), with \( \xi = 1 \) and \( N = 2 \). For this network, the NERs with the GQ, fLQ, and vLQs for \( \Lambda = 2^{-15}, 2^{-12}, \ldots , 2^{12}, 2^{15} \) are presented in Fig 4a. The horizontal and the vertical axes represent \( P \) in decibels (dBs), and the NER, respectively. We can observe
that both our GQ and vLQs achieve the maximal diversity $(2, -2)$, while the fLQ achieves diversity $(2, -4)$. Moreover, as we increase $\Lambda$, the array gain performance of our vLQs can be made arbitrarily close to that of the GQ.

![Graph showing performance results for a network with $K = R = L = 2$.](image)

The corresponding feedback rates of our vLQs are shown in Fig. 4b. The horizontal axis represents $P$ in decibels, while the vertical axis represents the feedback rate of the first (second) receiver in bits per channel state. Similarly, due to our choice of the network parameters, the feedback rates of each receiver will be the same. We can observe the validity of Theorem 2, as for any $\Lambda$, the required feedback rate decays to zero at high $P$. Also, by increasing $\Lambda$, the performance of the LQs can be made arbitrarily close to the one provided by the GQ, while still using very low feedback rates. As an example, at an NER of $10^{-5}$, vLQ-2$^{15}$ needs 1.25 bits per channel state per receiver on average and performs only 0.25dB worse than the GQ. At a SER of $10^{-5}$, vLQ-2$^{20}$ uses 0.65 bits, and GQ performs only 0.8dB better.

V. CONCLUSIONS AND DISCUSSIONS

We studied quantized beamforming in wireless relay-interference networks with any number of transmitters, receivers and amplify-and-forward (AF) relays. Our goal was to minimize the probability that at least one user incorrectly decodes its desired symbol(s). First, we introduced a generalized diversity measure in order to have a more precise description of the asymptotic performance of the network. It encapsulated the conventional measure as the first-order diversity. Additionally, it took into account the second-order diversity, which is concerned with the transmitter power dependent logarithmic terms that appear in the error rate expression. Then, using a relay-selection based global quantizer (GQ) and the localization method, we designed fixed-length and variable-length local quantizers (fLQs and vLQs). Our fLQ achieved maximal first-order diversity. Our vLQ provided not only maximal diversity gain, but also an array gain performance that can be made arbitrarily close to the one provided by the GQ. Moreover, it achieved all of its promised gains using arbitrarily low feedback rates, when the transmitter powers are sufficiently large.

REFERENCES