Abstract

This paper examines the rise in car use and decline in bus use in developing countries using a theoretical, mode choice model and numerical simulations. The empirical literature points to rising per capita income as a primary determinant of rising motor vehicle use, known as motorization. This analysis of commuter car/bus mode choice shows that in addition to rising income, other factors may drive rising car use at the urban level. First, greater income inequality increases car use if car use is still low, and reduces car use if it is already high. Second, traffic congestion hinders buses more than cars, causing positive feedback between car use and travel time that reduces bus use and contributes to its abrupt collapse. Third, policy interventions to reduce congestion in urban areas, such as tolling car use and reserving lanes for buses, increase consumer surplus by maintaining bus service as an alternate travel mode, even as incomes rise. Socially optimal reserved bus lanes may achieve most of the gains from a socially optimal toll on car use.

JEL Classification: O1; R4

Keywords: Motorization; Mode choice; Congestion; Urban growth; Developing countries

* I am highly indebted to my advisor Jan Brueckner for his guidance. I am grateful to Kenneth Small, Harris Selod, Volodymyr Bilotkach, and the editor and anonymous referees for their helpful comments and encouragement. I am also thankful to the Department of Economics and School of Social Sciences at UC Irvine, and the University of California Transportation Center for their generous financial support.

† University of California, Irvine, Department of Economics, 3151 Social Science Plaza, Irvine, CA 92697-5100, USA. E-mail address: kutzbach@uci.edu. Phone number: 773-936-5889.
1 Introduction

As some developing countries close the gap with developed countries in per capita income, they are also rapidly approaching the high rates of automobile use and urban traffic congestion characteristic of cities in developed countries. This congestion convergence is most apparent in megacities in developing countries, those with over 10 million inhabitants. For example, downtown weekday travel speeds average 10 kilometers per hour or less in Bangkok, Manila, Mexico City, and Shanghai, and 15 kilometers per hour or less in Kuala Lumpur and São Paulo (United Nations Centre for Human Settlement, 2002). The World Bank (2002) emphasizes the looming problem of traffic congestion for cities in developing countries, and suggests that it is likely to worsen as most developing countries presently have about 100 cars per 1,000 people, compared with over 400 cars per 1,000 people in developed countries.\(^1\) This paper develops a model incorporating income growth, income inequality, and car/bus mode choice to explain how motorization progresses and to show how congestion reduction policies can manage its pace and increase consumer surplus.

Researchers have explained some of the causes of motorization using empirical methods, often based on the analysis of aggregate data. These analyses find a positive relationship between motor vehicles per capita and income per capita, both across countries and cities and over time within countries and cities. For example, Ingram and Liu (1997) assemble panel data for 50 countries and 35 urban areas and find that vehicles per capita rises at the same rate as income per capita, with passenger cars increasing faster than commercial vehicles. Similarly, Dargay and Gately (1999) estimate income elasticities for national vehicle ownership rates. They find higher income elasticity for lower income

\(^1\)For a general discussion of motorization in developing countries, see Gakenheimer (1999). For a general discussion of increased car use due to increased population and affluence, longer and more numerous trips, increased labor force participation, and decentralized land use, see Giuliano (1998).
countries, and predict a rapid increase in car use in countries such as China, India, and Pakistan, where the growth rate of cars per capita is twice that of income per capita.

While these empirical estimates are useful for predicting national motorization trends, they do not address several important aspects of motorization at the urban level. First, empirical analyses of motorization often summarize the national income distribution with per capita income alone, leaving out measures of income inequality. Low and middle income developing countries have a wide range of inequality, which may persist even as overall income rises (Lopez and Servén, 2006). Studies in Brazil (Vasconcellos, 2005), India (Baker et al., 2005), and China (Liu, 2006), use individual level data to show that higher income commuters are more likely to travel by car. Thus, income inequality may be relevant for explaining motorization.\(^2\)

Second, although private cars may still have a lower commuter mode share than buses in many developing countries, as a share of motor vehicles, cars dominate the roads. For example, in Lima, Peru, 17 percent of motorized trips are by car and 83 percent by bus (UNCHS, 2002). However, 76 percent of vehicles are cars while only 18 percent are buses (Davis et al., 2004). In Nairobi, Kenya, 6 percent of commuters travel by car and 70 percent by bus. However, 88 percent of vehicles are cars while only 4 percent are buses. With frequent stops and less maneuverability, buses in cities of developing countries tend to be slower than cars, averaging 15 km per hour compared to around 25 km per hour for cars (Vasconcellos, 2001). This difference in speed gives commuters an incentive to switch to car travel, if they can afford it. The consequences of rising car use are apparent in the case of São Paulo, Brazil, where private auto use rose from a 26 percent mode share in 1967 to 47 percent in 1997 and bus use declined from 59 percent to 39 percent (Vasconcellos, 2005). The increase in traffic volume has led to congestion over 80 percent

\(^2\)Gakenheimer (1999) finds that average income of the top quintile is also strongly associated with motorization.
of the road system and a drop in average car speeds from 28 km/hr to 17 km/hr from the 1980s to the 1990s. Thus, mode choice contributes to the overall rise in motor vehicles per capita.

Third, Ingram and Liu (1997) find that the size of urban road networks in developing countries is limited by a saturation point, so that rising car use eventually results in congestion. As an alternative to expanding road capacity, some cities are turning to congestion reduction policies including road pricing and reserved bus lanes. Both policies are associated with reduced congestion and increased bus mode share (World Bank, 2002).

The model of motorization presented here addresses these three topics by building on the theoretical mode choice literature, with a focus on the choice of car and bus travel. In a common setup of the mode choice problem (Arnott and Yan, 2000), commuters travel from a fixed origin to a fixed destination using one of the separate modes or routes available. Mode choice analysis often focuses on optimal pricing (Glazer and Niskanen, 2000; Small and Yan, 2001; Kraus, 2003; Armelius, 2005). Since the present model focuses on the transition from bus use to car use, the choices are car travel, bus travel, and no travel. Because cars and buses typically share the same roadway in developing countries, use of each mode directly affects the travel time of both modes.\(^3\)

Modeling car/bus mode choice also requires accounting for economies of density in bus transport. Mohring (1972) shows that increasing bus frequency reduces wait time for bus users, thereby reducing the average cost of bus transport. With lower cost in terms of wait time, bus ridership will increase. It is then socially optimal for a bus provider to further increase bus service and achieve an economy of scale. This positive feedback between ridership and the socially optimal level of bus service is a “virtuous

\(^3\)The model does not incorporate urban rail travel, which is often unavailable in developing countries, and when present, limited to a few portions of the city. The model also omits non-motorized travel, which is most common for short distance trips, but the model does allow consumers to refrain from commuting.
circle.” Although the Mohring model is based on a publicly run bus system, a fragmented and informal bus transport system, as is present in many developing countries, may also respond to changes in ridership by increasing service (Cervero, 2000).

The present analysis is not the first to model car/bus mode choice, but it takes a more general approach appropriate for modeling motorization. Researchers often use models of car/bus mode choice to predict the effects of policy changes for a narrow range of parameters in a specific city of a developed country (Mohring, 1979; Small, 1983; Van Dender, 2003; Small, 2004; Wichiensin et al., 2007). The present analysis is not meant to precisely predict transport outcomes, but to demonstrate the sensitivity of equilibrium mode use to underlying parameters as may be typical in a developing country. In developing countries, income growth and income inequality interact with the virtuous circle of bus service, resulting in bus service thriving or collapsing. Mohring (1972) predicts that, “as real incomes increase, the demand for these services will undoubtedly continue to decline.” Indeed, bus ridership now has a small mode share in many developed countries, even with subsidies. In contrast, a large share of commuters in many developing countries, with lower incomes and high population densities, still rely on bus transport. Whether developing countries can maintain a high bus mode share will depend on the characteristics of those countries and the pursuit of policy interventions that reduce congestion.

This paper contributes to the understanding of motorization in three ways. First, it formally shows how car use depends on income inequality as well as overall income growth. While high inequality is likely to stimulate motorization at low overall income levels, it will depress car use for high overall income levels. Second, because of positive feedback between car use and congestion, motorization in urban areas is likely to be highly non-linear. Third, charging car users a toll or reserving road capacity for bus use will shift mode share to bus use, reduce in-vehicle travel time for both cars and buses, and
increase consumer surplus. A car use toll that is large enough to make bus use sustainable benefits from the virtuous circle, and increases consumer surplus even without considering revenue from the toll. Reserving even a small portion of road capacity for buses may approximate the consumer surplus gains from a socially optimal car use toll. Thus, congestion reduction policies should have broad-based support.

The organization of the paper is as follows. Section 2 presents a model of car/bus mode choice with equilibrium conditions, stability analysis, and relevant comparative statics. Section 3 illustrates the model with simulations. Section 4 considers policy interventions to reduce congestion. Section 5 concludes.

2 Model

The present analysis uses a stylized model of equilibrium car/bus mode use with heterogeneous users. The total number of commuters is exogenous in the short run but endogenous in the long run (commute distance, road capacity, and other parameters remain exogenous).

2.1 Travel time submodel

This subsection describes the mechanical effect of mode use on travel time, treating mode use as exogenous. Suppose there are $\bar{N}$ consumers, who are potential commuters. Commuters travel from a place of residence to a place of work along a shared roadway by one of several modes, with mode $j$ having $N_j$ commuters and travel time $T_j$. Assume that the number of consumers is large, so functions of $N_j$, introduced below, may be treated as continuous and differentiable.

As discussed earlier, the present analysis adapts the two-mode model for a car/bus system where $j = c, b$ and $N_c + N_b = N \leq \bar{N}$ (not all consumers need commute).
Because cars and buses share road space, the number of commuters using both modes will directly affect in-vehicle travel time of a given mode, denoted $T_{v_j}^c(N_j, N_{-j})$. Therefore, $\partial T_{v_j}^c / \partial N_j > 0$ and $\partial T_{v_j}^c / \partial N_{-j} > 0$. Bus travel time also includes wait time, $T_{w_b}^b(N_b, T_{v_b}^b)$, which depends on total ridership and bus in-vehicle travel time. Assuming that the number of buses increases with the number of bus users, an increase in bus riders directly reduces wait time, so $\partial T_{w_b}^b / \partial N_b < 0$. Meanwhile, greater in-vehicle bus travel time corresponds with slower circulation and increases bus wait time, so $\partial T_{w_b}^b / \partial T_{v_b}^b > 0$.

Total travel times for the car and bus modes are then

$$T_c(N_c, N_b) \equiv T_{v_c}^c(N_c, N_b),$$  
(1)

$$T_b(N_b, N_c) \equiv T_{v_b}^b(N_b, N_c) + T_{w_b}^b(N_b, T_{v_b}^b(N_b, N_c)).$$  
(2)

While increased use of either mode increases car travel time, bus travel time, composed of in-vehicle and wait time, responds to both congestion and economies of density, so that

$$\frac{\partial T_b}{\partial N_c} = \frac{\partial T_{v_b}^b}{\partial N_c} \left(1 + \frac{\partial T_{w_b}^b}{\partial T_{v_b}^b} \right) > 0,$$  
(3)

$$\frac{\partial T_b}{\partial N_b} = \frac{\partial T_{v_b}^b}{\partial N_b} \left(1 + \frac{\partial T_{w_b}^b}{\partial T_{v_b}^b} \right) + \frac{\partial T_{w_b}^b}{\partial N_b}.$$  
(4)

Increased car use (3) unambiguously increases bus in-vehicle time and wait time due to congestion. In contrast, an increase in bus use (4) results in more buses and less wait time, so that bus travel time may decrease (if there are relatively few buses initially).\(^4\)

The relative magnitudes of the effects of car and bus use on in-vehicle time of each

\(^4\)The present model assumes that buses are always at capacity and does not consider the positive effect of bus use on boarding and unloading time or discomfort of other passengers, treated explicitly in Kraus (1991); thus, economies of density in wait time are dominant.
mode are known. Because cars take up more road space per user than buses, the marginal contribution of car users to congestion and travel time on either mode is greater, with

$$\frac{\partial T^v_j}{\partial N_{c_j}} > \frac{\partial T^v_j}{\partial N_{b_j}}.$$  

(5)

Because buses are larger, less maneuverable, and may require curb access, higher traffic volume is likely to increase bus travel time more than car travel time, so that

$$\frac{\partial T^v_b}{\partial N_{j}} > \frac{\partial T^v_c}{\partial N_{j}},$$  

(6)

and $$T^v_b(N_b, N_c) > T^v_c(N_c, N_b)$$ holds. McKnight et al. (2004) describe empirical studies in the United States that show bus travel time to be proportionally greater than car travel time. In developing countries, bus speeds are also typically slower than car speeds in congested urban areas (Vasconcellos, 2001).

Based on these assumptions, a transfer of commuters from buses to cars, as happens in motorization, increases travel times for both modes (although travel time could decrease for the commuters switching to car use). With total usage fixed at $$N_c + N_b = N,$$

$$\frac{\partial T^c_c}{\partial N_{c_j} |_{N}} \equiv \left. \frac{\partial T^c_c}{\partial N_{b_j}} \right|_{N} = \left. \frac{\partial T^v_c}{\partial N_{c_j}} - \frac{\partial T^v_c}{\partial N_{b_j}} \right|_{N} > 0,$$

(7)

$$\frac{\partial T^c_b}{\partial N_{c_j} |_{N}} \equiv \left. \frac{\partial T^c_b}{\partial N_{b_j}} \right|_{N} = \left( \left. \frac{\partial T^v_b}{\partial N_{c_j}} - \frac{\partial T^v_b}{\partial N_{b_j}} \right|_{N} \right) \left( 1 + \frac{\partial T^w_c}{\partial T^v_c} \right) - \frac{\partial T^w_b}{\partial N_b} > 0.$$  

(8)

Furthermore, based on (6), rising traffic volume increases bus in-vehicle time more than car in-vehicle time, so that (8) is strictly greater than (7).
2.2 Utility

A consumer’s utility from a trip equals trip value minus the money and time cost of commuting. For simplicity, the money costs of car and bus travel, $c_c$ and $c_b$, do not depend on travel time. To reflect the relatively high cost of using a car, the analysis assumes that $c_c > c_b$. Note that the model does not treat the vehicle ownership decision as a separate, fixed cost.

The time cost of travel increases with value of time and travel time. Suppose consumer value of time is proportional to consumer wage (Brownstone and Small, 2005). A commuter on mode $j$ with hourly income $Y$ and value of time $vY$ ($v > 0$) would have time cost $vYT_j$. Furthermore, assume that consumer trip value is also proportional to income. Specifically, a consumer values a trip at a proportion $h > 0$ of hourly income, so that trip value is $hY$. If consumer incomes are heterogeneous, then consumer time cost and trip value will be heterogeneous as well. The function $Y(i)$ specifies income, with $i$ indexing consumers by decreasing income from 1 to $\bar{N}$, so that $\partial Y/\partial i < 0$.

The utility of commuter $i$ traveling on mode $j$ with $N_c$ car commuters, $N$ total commuters (giving $N - N_c$ bus commuters), and parameter set $\Theta$ (including $v$, $h$, $\bar{N}$, and other parameters introduced later), is denoted $V_j(i, N_c, N, \Theta)$. With this general formulation, commuter $i$’s utility for traveling by car and by bus is

$$V_c(i, N_c, N, \Theta) = Y(i)[h - vT_c(N_c, N - N_c)] - c_c, \quad (9)$$

$$V_b(i, N_c, N, \Theta) = Y(i)[h - vT_b(N - N_c, N_c)] - c_b. \quad (10)$$

A consumer who does not travel has utility zero. Figure 1 (discussed in greater detail below) displays $V_c$ and $V_b$ as functions of the index, $i$, where $h - vT_c(N_c, N - N_c) > 0$ and $h - vT_b(N - N_c, N_c) > 0$ hold so that trip value is greater than the time cost of travel.

---

5See Glazer and Niskanen (2000) and the articles they reference.
for all consumers. Because lower income consumers (higher \(i\)) gain less from commuting, \(\partial V_c/\partial i < 0\) and \(\partial V_b/\partial i < 0\) hold.

### 2.3 Equilibrium mode use

In the short run, where the number of commuters is fixed, a commuter maximizes utility by choosing the mode with the lowest travel cost. The cost difference for commuter \(i\) is denoted \(\Delta(i, N_c, N, \Theta) = V_c(i, N_c, N, \Theta) - V_b(i, N_c, N, \Theta)\), or

\[
\Delta(i, N_c, N, \Theta) = Y(i)v[T_b(N - N_c, N_c) - T_c(N_c, N - N_c)] - (c_c - c_b),
\]

where the first term indicates the time cost savings from car travel and the second term indicates the money cost premium. Commuter \(i\) will prefer car travel if \(\Delta(i, N_c, N, \Theta) > 0\), bus travel if \(\Delta(i, N_c, N, \Theta) < 0\), and will be indifferent if \(\Delta(i, N_c, N, \Theta) = 0\). Because \(T_b > T_c\) (making the first term positive) and \(Y\) is decreasing in \(i\), \(\partial \Delta/\partial i < 0\) holds (making \(V_c\) steeper than \(V_b\) in Figure 1). Thus, if consumer \(i'\) prefers car travel, then all consumers with \(i < i'\), with even greater values of time, will also prefer car travel. Conversely, if consumer \(i'\) prefers bus travel, then all consumers with \(i > i'\), with even lesser values of time, will also prefer bus travel. An equilibrium exists when no consumer would prefer to switch modes.

In a short-run equilibrium, a consumer \(i^*\) is indifferent between car and bus travel, so that all consumers with \(i < i^*\) prefer car travel and all consumers with \(i > i^*\) prefer bus travel. Given the indexing of consumers, the equilibrium number of car commuters, denoted \(N^*_c\), is equal to the index of the indifferent marginal car commuter, so that \(i^* = N^*_c\). In Figure 1, the equilibrium marginal car commuter marks the intersection of the \(V_c\) and \(V_b\) curves.

For convenience, the equilibrium analysis will use the function \(I(N_c)\) to denote the
index of the marginal car commuter, where \( i = I(N_c) \equiv N_c \). The car use equilibrium condition, using equation (11) and the \( I \) function, is

\[
\Delta(I(N_c^*)), N_c^*, N, \Theta) = 0. \tag{12}
\]

Equation (12) is equivalent to \( \Delta(N_c^*, N_c^*, N, \Theta) = 0 \).

In a long-run equilibrium, the total number of commuters, \( N \), is also endogenous. Determining the equilibrium value of this variable, denoted \( N^* \), requires an additional condition involving the individual who is indifferent between bus commuting and not commuting at all. For this individual, who has the lowest income among bus commuters and whose index is thus \( N^* \), the utility of bus commuting is zero (and is thus equal to the utility of not commuting). Adding this condition to equation (12) and replacing \( N \) with \( N^* \), the long-run equilibrium conditions are

\[
\Delta(I(N_c^*)), N_c^*, N^*, \Theta) = 0, \tag{13}
\]
\[
V_b(I(N^*), N_c^*, N^*, \Theta) = 0. \tag{14}
\]

In Figure 1, the long-run equilibrium marginal commuter marks where the \( V_b \) curve reaches a level of zero. It should be noted, however, that in an equilibrium with only car commuters, \( N^* \) and \( N_c^* \) are the same and determined by the intersection of the \( V_c \) curve and the zero line (the \( V_b \) curve must then lie below this intersection).\(^6\)

\(^6\)The long-run model may also be extended to include non-motorized walking or bicycling transport as a third option, with constant travel time and no effect on traffic volume.
2.4 Stability of equilibrium

To analyze stability of the short-run equilibrium, define

\[ \Gamma(N_c, N, \Theta) \equiv \Delta(I(N_c), N_c, N, \Theta), \tag{15} \]

as the marginal car commuter cost-difference function. The equilibrium condition can be written \( \Gamma(N_c^*, N, \Theta) = 0 \).

An equilibrium is stable if a slight perturbation in the index of the marginal car commuter away from \( i = N_c^* \) results in a return to the same \( N_c^* \). Consider an equilibrium where \( \partial \Gamma / \partial N_c < 0 \). With a reduction in car use to \( N_c^* - \epsilon \), the new marginal car commuter faces a cost difference \( \Gamma(N_c^* - \epsilon, N, \Theta) > 0 \), and will strictly prefer car travel. Thus, the equilibrium will be re-established as commuters with \( i > N_c^* - \epsilon \) return to car use. In the same way, an increase in car use to \( N_c^* + \epsilon \) presents the new marginal car commuter with cost difference \( \Gamma(N_c^* + \epsilon, N, \Theta) < 0 \), dissuading the commuter from car travel and re-establishing the equilibrium \( N_c^* \). Thus, the equilibrium is stable when \( \partial \Gamma / \partial N_c < 0 \).

In contrast, consider an equilibrium with \( \partial \Gamma / \partial N_c > 0 \). With a reduction in car use to \( N_c^* - \epsilon \), the cost difference is \( \Gamma(N_c^* - \epsilon, N, \Theta) < 0 \), and the new marginal car commuter will prefer bus travel, leading to further decline in car use. An increase in car use to \( N_c^* + \epsilon \) gives a cost difference \( \Gamma(N_c^* + \epsilon, N, \Theta) > 0 \), encouraging even more car use. Thus, the equilibrium is unstable when \( \partial \Gamma / \partial N_c > 0 \).

To see when \( \Gamma \) is increasing or decreasing with \( N_c \), consider the partial derivative

\[ \frac{\partial \Gamma}{\partial N_c} = \frac{\partial \Delta}{\partial i} \frac{\partial i}{\partial N_c} + \frac{\partial \Delta}{\partial N_c}. \tag{16} \]

Using equation (11), the first term is negative because the time cost savings are smaller
for lower income (higher index) commuters and because $\partial I / \partial N_c = 1$. Because rising car use increases the time savings from car travel (see equations (7) and (8)), the second term is positive.

Based on the following assumptions for the income distribution and travel time, Figure 2.a illustrates the marginal car commuter cost-difference function. Assume an upward skewed income distribution with lower density at higher incomes (lower index), so that the magnitude of the first term in (16) decreases with $N_c$. In addition, assume that in-vehicle time is convex in traffic volume, so that the magnitude of the second term in (16) increases with $N_c$. As shown in Figure 2.a, for low car use, the first term in (16) dominates and $\partial \Gamma / \partial N_c < 0$. For a higher car use, where congestion and bus wait time are substantially higher, the second term in (16) dominates, resulting in $\partial \Gamma / \partial N_c > 0$. As the number of bus commuters falls to zero, bus waiting time and time cost savings approach infinity. With these assumptions $\partial^2 \Gamma / \partial N_c^2 > 0$.

Given the properties of the $\Gamma$ function, the number of equilibria depends on the level of the function, which in turn depends on the parameter values (discussed in detail in the comparative statics section). For moderate parameter levels, $\Theta_M$, two interior equilibria exist. The left-most equilibrium, where $\partial \Gamma / \partial N_c < 0$, is stable. The right-most equilibrium, where $\partial \Gamma / \partial N_c > 0$, is unstable. For parameter values $\Theta_H$ leading to a higher $\Gamma$ function, no equilibrium exists and universal car use, a corner solution, prevails. For $\Theta_L$, one unstable equilibrium exists. See Figure 2.a.

The stable, interior equilibrium is of particular interest because it describes the prevailing mode split in many cities of developing countries, where bus use still accounts for a major share of commuting. While the equilibrium number of car commuters is resilient to modest, day-to-day perturbations, changes to elements of $\Theta$, such a shift in income or the cost of car travel, may increase or decrease equilibrium mode use. A large shift in $\Theta$ may result in a corner solution, universal bus or car use.
The stability criterion for the long-run equilibrium applies to an equation system rather than a single equation condition. Defining the marginal commuter cost-difference function

$$\Psi(N^*_c, N, \Theta) \equiv V_b(I(N), N^*_c, N, \Theta),$$

(17)

de the long-run equilibrium conditions can be written $\Gamma(N^*_c, N^*, \Theta) = 0$ and $\Psi(N^*_c, N^*, \Theta) = 0$.

As with $\Gamma$ for perturbations in $N_c$, $\Psi$ is stable for perturbations in $N$ when $\partial \Psi / \partial N < 0$ and unstable when $\partial \Psi / \partial N > 0$. To see where $\Psi$ is increasing or decreasing with $N$, for a given $N^*_c$, consider the derivative

$$\frac{\partial \Psi}{\partial N} = \frac{\partial V_b}{\partial i} \frac{\partial I}{\partial N} + \frac{\partial V_b}{\partial N}. \tag{18}$$

The first term in (18) is negative if the time cost of travel is less than trip value, because $\partial Y / \partial i < 0$ holds, as noted previously. The second term is positive or negative depending on traffic volume and the number of bus commuters, $N - N_c$. As shown in equation (4), increased bus use (holding car use constant) increases in-vehicle travel time (with more congestion) and reduces wait time (with more buses).

Where there are few bus users, bus time cost is greater than trip value so the first term in (18) will be positive. Because any ridership increase dramatically reduces waiting time, the second term is also positive and $\partial \Psi / \partial N > 0$, as shown in Figure 2.b. Where there are many bus users and wait time is lower, trip value is greater than time cost and the first term is negative. Where the economy of density in wait time is exhausted, the congestion effect will dominate and the second term will also be negative, resulting in $\partial \Psi / \partial N < 0$. In Figure 2.b, the left-most equilibrium is unstable to perturbations in $N$ and the right-most equilibrium is stable.
The above discussion is only a partial analysis. Long-run stability of the equilibrium requires that $\Gamma$ and $\Psi$ are each stable to perturbations in both $N_c$ and $N$. A sufficient condition for the equilibrium to be stable is that (16) and (18) are both negative, and that bus use sufficiently exhausts economies of density.\footnote{If the conditions $\partial \Gamma / \partial N_c + \partial \Psi / \partial N < 0$ and $\partial \Gamma / \partial N_c \cdot \partial \Psi / \partial N - \partial \Gamma / \partial N \cdot \partial \Psi / \partial N_c > 0$ hold, an equilibrium is locally, asymptotically stable. As stated above, $\partial \Gamma / \partial N_c < 0$ and $\partial \Psi / \partial N < 0$ are sufficient to satisfy the first condition. Because $\partial \Psi / \partial N_c < 0$ always holds (see (8)), $\partial \Gamma / \partial N_c < 0$ and $\partial \Psi / \partial N < 0$ along with bus use sufficiently exhausting economies of density (so that (4) is positive and $\partial \Gamma / \partial N > 0$ holds), are sufficient to satisfy the second condition, but not necessary. The derivation of this result is available upon request.} The comparative statics subsection will focus on the short-run equilibrium while numerical simulations will explore both the short and long-run equilibrium results.

2.5 Functional forms

Specific functional forms for car and bus in-vehicle time, bus wait time, and income in this simplified model help to illustrate comparative statics and are a basis for numerical simulations. The capacity restraint function (Bureau of Public Roads, 1964) specifies in-vehicle time, $T^v$, as a function of free-flow travel time, $t_0 > 0$, traffic volume (the number of cars attempting to use the road per hour), road design capacity, $k_r > 0$ (the number of cars that may use the road per hour with a minimal reduction in speed), and two parameters, $\alpha > 0$ and $\beta > 0$, that modify the volume-to-capacity ratio as a multiplier and exponent respectively. The capacity restraint function is then

$$T^v(\text{volume}) = t_0 \left(1 + \alpha \left(\frac{\text{volume}}{k_r}\right)^\beta\right). \tag{19}$$

Let one bus contribute the equivalent of $e_b > 1$ single-occupant passenger cars to traffic volume. Also assume that buses are always at capacity $k_b > 1$, so that total ridership divided by bus capacity gives the number of buses, $(N - N_c) / k_b$. Recalling
the assumption that bus users contribute less to traffic volume than car users (5), let $e_b/k_b < 1$ hold. Car in-vehicle time is then given by

$$T_c(N_c, N - N_c) = t_0 \left( 1 + \alpha \left( \frac{N_c + e_b(N - N_c)/k_b}{k_r} \right)^{\beta} \right).$$  \hspace{1cm} (20)

Because buses are slower and less maneuverable, assume that bus in-vehicle time is $t_b T_c$, with $t_b > 1$.

Bus headway equals the in-vehicle time for a bus, $t_b T_c$, divided by the number of buses traveling the route, $(N - N_c)/k_b$. To illustrate this point, if bus in-vehicle time is 60 minutes and there are two buses, then bus headway is 30 minutes. If bus arrivals are uncoordinated and uniformly distributed, as is likely with an informal system, expected waiting time is equal to half the headway, or 15 minutes in this example (Mohring, 1972).

Lastly, empirical evidence (Wardman, 2001) indicates that consumers value waiting time more than in-vehicle time. The parameter $w \geq 1$ reflects the relative valuation of waiting time to in-vehicle time. Expected, effective wait time is then $w(1/2)t_b T_c/(N - N_c)/k_b$.

With the addition of wait time to bus in-vehicle time, expected bus travel time is

$$T_b(N - N_c, N_c) = t_b T_c(N_c, N - N_c)[1 + w k_b/2(N - N_c)].$$ \hspace{1cm} (21)

These specific functional forms for travel time exhibit the same characteristics as the general forms derived in equations (1) through (8) above.

With substitution of the travel time functions (20) and (21), $\Gamma$, the marginal car

---

8Following McKnight et al. (2004) as well as Wichiensin et al. (2007). If $t_b = 1$, the transition of $\Gamma$ from a downward to an upward slope will depend only on wait time, occur at a higher index, and be more abrupt.
The commuter cost-difference function, is rewritten as

\[
\Gamma(N_c, N, \Theta) = Y(N_c)vt_0 \left(1 + \alpha \left(\frac{N_c + e_b(N - N_c)/k_b}{k_r}\right)^\beta\right) \\
\cdot \left[(t_b - 1) + \frac{t_b w k_b}{2(N - N_c)}\right] - (c_c - c_b).
\]

(22)

Because the scale and spread of income distributions are highly varied among developing countries, comparative statics and numerical simulation require a functional form for \(Y(i)\) where these two characteristics may vary independently. In a study using income quintile data from developed and developing countries, Lopez and Servén (2006) find that a two parameter, lognormal density describes income distributions well, with a high density portion representing the middle class and a long, low-density, upper tail representing the very rich.

Let income be a lognormal distributed variable defined \(Y = \exp(X)\), where \(X\) is distributed normal with mean \(\mu\) and standard deviation \(\sigma\). \(Y(i)\) is defined implicitly by an integral of the lognormal density

\[
\frac{\tilde{N} + g - i}{\tilde{N} + g} = \int_0^{Y(i)} \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(\ln y - \mu)^2}{2\sigma^2}\right) dy \equiv F(Y(i)|\mu, \sigma),
\]

(23)

where \(g\) is chosen to yield reasonable values for the highest and lowest incomes. Note that when \(i = \tilde{N}\) (lowest income consumer), the cumulative distribution function value equals \(g/(\tilde{N} + g) > 0\), so that \(Y(\tilde{N}) > 0\), and when \(i = 1\) (highest income consumer), the cdf value equals \((\tilde{N} + g - 1)/(\tilde{N} + g) < 1\), so that \(Y(1) < \infty\). The simulation section uses \(g = 1\). Consumer \(i\)'s income is then given by the inverse cdf

\[
Y(i) = F^{-1}\left((\tilde{N} + g - i)/(\tilde{N} + g)|\mu, \sigma\right).
\]

(24)
Consider the effect of an increase in $\mu$, with $\mu_1 > \mu_0$ and $X_1 \sim N(\mu_1, \sigma)$, $X_0 \sim N(\mu_0, \sigma)$. It is a property of the normal distribution that $X_1 = X_0 + (\mu_1 - \mu_0)$. Transforming this property to a lognormal distribution gives $Y_1 = Y_0 \exp(\mu_1 - \mu_0)$. Thus, increasing $\mu$ scales up the entire income distribution, analogous to overall growth.

Consider the effect of an increase in $\sigma$, with $\sigma_1 > \sigma_0$ and $X_1 \sim N(\mu, \sigma_1)$, $X_0 \sim N(\mu, \sigma_0)$. It is a property of the normal distribution that $X_1 = \sigma_1(X_0 - \mu)/\sigma_0 + \mu$. Transforming this property to a lognormal distribution gives $Y_1 = \exp(\sigma_1(\ln Y_0 - \mu)/\sigma_0 + \mu)$. Increasing $\sigma$ increases incomes for those above median income ($i < \bar{N}/2$, where $\ln Y_0(i) > \mu$ holds) and decreases incomes for those below ($i > \bar{N}/2$, where $\ln Y_0(i) < \mu$ holds), analogous to rising inequality. For the range $\sigma = 0.5$ to $\sigma = 1$, the degree of inequality in the lognormal inverse cdf corresponds well with observed inequality as measured by the Gini index (World Bank, 2007).

### 2.6 Comparative statics

Comparative static analysis of the short-run, stable, interior equilibrium shows how car use responds to exogenous parameter changes and suggests relationships that may be important for motorization. Simulations will further illustrate some of these comparative statics, including long-run results.

First consider the effect of free-flow travel time, $t_0$, on equilibrium car use, $N_c^*$. Higher free-flow travel time, which may suggest greater commute distance if free-flow speed is unchanged, affects commuter mode choice by increasing the time cost of travel. Using the equilibrium condition $\Gamma(N_c^*, N, \Theta) = 0$ and totally differentiating the marginal car commuter cost-difference function, $\Gamma$ (equation (22)), with respect to $N_c^*$ and $t_0$ (an element of $\Theta$) yields

$$\frac{\partial N_c^*}{\partial t_0} = -\frac{\partial \Gamma}{\partial t_0} \frac{\partial \Gamma}{\partial N_c^*} > 0.$$  \hfill (25)
The inequality follows because \( \partial \Gamma / \partial N_c < 0 \) holds in the stable equilibrium and because
\[
\frac{\partial \Gamma}{\partial t_0} \simeq \frac{\partial T_c}{\partial t_0} > 0,
\] (26)
where “\( \simeq \)” signifies “same sign” and where the inequality follows from inspection of (22).

Given the proportionality between car and bus in-vehicle time, higher in-vehicle car travel time, \( \partial T_c / \partial t_0 > 0 \), increases the time cost of bus travel more than car travel for the marginal car commuter, so \( \partial \Gamma / \partial t_0 > 0 \) holds. The marginal car commuter cost-difference curve in Figure 2.a therefore rises, attracting more car users. This result is consistent with evidence from microdata in developing countries (Vasconcellos, 2001; Baker et al., 2005), indicating that commuters traveling long distances (those with higher \( t_0 \)’s) are more likely to travel by car (not accounting for the endogeneity of commute distance).

To show short-run comparative statics for other parameters \( \theta \) (elements of \( \Theta \)), only the relevant portion of the partial derivatives \( \partial \Gamma / \partial \theta \) need be displayed. Because \( \partial \Gamma / \partial N_c^* < 0 \) always holds at the stable equilibrium, \( \partial N_c^* / \partial \theta \simeq \partial \Gamma / \partial \theta \).

For the lognormal density income distribution, an increase in the income scale \( \mu \) increases time cost savings from car travel for the marginal car commuter, and thereby increases equilibrium car use. Using (22) and (24), the partial derivative showing this effect is
\[
\frac{\partial N_c^*}{\partial \mu} = - \frac{\partial \Gamma / \partial \mu}{\partial \Gamma / \partial N_c^*} \simeq \frac{\partial Y(N_c^*)}{\partial \mu} > 0.
\] (27)
The magnitude of the effect depends on the density of the income distribution and degree of congestion at \( N_c^* \) (illustrated by Figure 3 in the simulation section).

To understand the forces determining the magnitude of (27), consider first a model without travel time feedback, where travel time is constant regardless of the level of car use. The second term of equation (16) then drops out and \( \partial \Gamma / \partial N_c^* = \partial \Delta / \partial i < 0 \). The
magnitude of $\partial \Delta / \partial i$ depends on the density of the income distribution at $N_c^*$, with a steeper income gradient at low density portions. When car use is low, $N_c^*$ will be in the low density portion of the lognormal income distribution and the marginal car commuter will have a relatively high income. Thus, both $\partial Y/\partial \mu$ and $\partial \Delta / \partial i$ will be relatively large in magnitude for the marginal car commuter, so that the ratio expression in (27) results in a moderate increase in car use. Contrast this case with a rise in income scale when car use is already widespread and $N_c^*$ is in the high density, median income portion of the lognormal distribution. The increase in car use will then be greater because $\partial Y/\partial \mu$ is moderate while $\partial \Delta / \partial i$ is near its smallest point. For a high $N_c^*$, where income and density are both low, the increase in car use will be moderate again.

Returning to the model where rising car use increases travel time, the second term in equation (16), $\partial \Delta / \partial N_c > 0$, increases in magnitude with $N_c^*$. Thus, for $N_c^*$ near the median income, rising congestion will further increase travel time savings and reduce the magnitude of $\partial \Gamma / \partial N_c^* < 0$, resulting in an even larger magnitude $\partial N_c^* / \partial \mu > 0$. For a low income (high index) marginal car commuter, high congestion may lead to $\partial \Gamma / \partial N_c^* = 0$, and a rise in $\mu$ would result in universal car use (as depicted by $\Theta_H$ in Figure 2.a.)

In summary, this comparative static analysis suggests that subsequent increases in income scale should cause greater increases in car use due to the lognormal density of the income distribution and feedback from congestion, culminating in a corner solution of universal car use when congestion and wait time rise sufficiently to make bus travel unsustainable. This chronology corresponds with observed patterns of rapid motorization in growing urban areas of developing countries.

The effect of rising income inequality, $\sigma$, on modal split is more nuanced, increasing car use if equilibrium car use is low, and decreasing car use if equilibrium car use is high.
The relevant partial derivative is

\[ \frac{\partial N^*_c}{\partial \sigma} \approx \frac{\partial \Gamma}{\partial \sigma} \approx \frac{\partial Y(N^*_c)}{\partial \sigma} \begin{cases} > 0, & N^*_c < \tilde{N}/2 \\ < 0, & N^*_c > \tilde{N}/2 \end{cases} \tag{28} \]

If less than half of consumers use a car, then an increase in inequality raises the income of the marginal car commuter, increasing the commuter’s time cost savings from car travel and raising equilibrium car use. Conversely, if more than half use a car, then an increase in inequality lowers the income of the marginal car commuter, reducing the commuter’s time cost savings from car travel and lowering equilibrium car use. The economic intuition of equation (28) is that for a low income scale with low car use, high inequality expands the high income, high time-value group that will travel by car. For a high income scale with high car use, high inequality expands the low income, low time-value group that will not travel by car.\(^9\)

An increase in the total population of potential road users, \(\tilde{N}\), also increases equilibrium car use (assuming \(\mu\) and \(\sigma\) remain constant). The partial derivative is

\[ \frac{\partial N^*_c}{\partial \tilde{N}} \approx \frac{\partial \Gamma}{\partial \tilde{N}} \approx \frac{\partial Y(N^*_c)}{\partial \tilde{N}} > 0, \tag{29} \]

where \(\partial Y(N^*_c)/\partial \tilde{N} > 0\), because an increase in population increases the income to which the marginal car commuter’s index corresponds. The resulting increase in travel time savings for commuter \(i = N^*_c\) leads to greater car use. As with the discussion of the income scale comparative static effect, traffic congestion feedback will compound the

\(^9\)There has been little empirical research on the effect of inequality on motorization. One study that does consider the effect of inequality (York, 2003) regresses the motorization rate on per capita income, income inequality and other variables and finds no significant effect for inequality among middle income countries (measured by the Gini index). An empirical finding of no significant linear effect would be consistent with the comparative static result that the sign of the effect of inequality depends on whether car use would otherwise be low or high.
effect of population growth and further increase car use. Thus, population growth in megacities of developing countries with fixed road infrastructure will lead to a higher level and rate of car use, even without income growth.

An increase in road capacity, $k_r$, reduces car use in the short run. The partial derivative is

$$\frac{\partial N^*_c}{\partial k_r} \simeq \frac{\partial \Gamma}{\partial k_r} \simeq \frac{\partial T_c}{\partial k_r} < 0. \tag{30}$$

Increasing capacity reduces congestion and lowers travel time on both modes, so that $\partial T_c/\partial k_r < 0$ (see (22)). Reduced travel time induces more commuters to travel by bus because bus in-vehicle travel time falls by more than car in-vehicle travel time and because bus wait time falls. The shift to bus use will further reduce congestion and travel time in a virtuous circle.

This result of increased capacity reducing travel time differs from the standard Downs-Thomson result (Down, 1962; Thomson, 1977), where an increase in road capacity may, paradoxically, increase travel time due to public transit users’ latent demand for road travel.\(^\text{10}\) In the present car/bus model, however, interdependent congestion and the assumption of bus in-vehicle travel time being proportionately greater than car in-vehicle travel time drive the result of capacity expansion reducing travel time for both modes.\(^\text{11}\)

Other comparative static results are mostly intuitive based on equation (22). Parameter shifts that increase the travel time savings of car travel increase equilibrium car use. Thus,

$$\frac{\partial N^*_c}{\partial t_b} > 0, \quad \frac{\partial N^*_c}{\partial e_b} > 0, \quad \frac{\partial N^*_c}{\partial v} > 0, \quad \frac{\partial N^*_c}{\partial w} > 0, \quad \frac{\partial N^*_c}{\partial \alpha} > 0, \quad \frac{\partial N^*_c}{\partial \beta} > 0. \tag{31}$$

\(^{10}\)For the standard Downs-Thomson result, identical commuters choose between a fast route that is congestible (road) and a slow route that has economies of scale (rail).

\(^{11}\)Downs (1962), Mohring (1979), and Takeuchi (1999) discuss this potential outcome.
Bus capacity has an ambiguous net effect on car use, with

$$\frac{\partial N^*_c}{\partial k_b} \simeq \frac{\partial T^*_c}{\partial k_b} = Y(N^*_c) v \left( \frac{\partial T^*_c}{\partial k_b} \left[ (t_b - 1) + \frac{t_b w k_b}{2(N - N^*_c)} \right] + \frac{T_c t_b w}{2(N - N^*_c)} \right),$$

(32)

where (32) is positive if the positive effect of fewer buses on wait time dominates the negative effect of less congestion. Lastly, $\partial N^*_c / \partial c_c < 0$ and $\partial N^*_c / \partial c_b > 0$.

3 Simulation

With scarce data on how mode split and congestion vary over time within and between cities of developing countries, empirical analysis to verify these comparative statics may not be possible. Rather, a numerical simulation of mode choice with plausible parameter values illustrates the causes of motorization, its consequences, and the likely impact of congestion reduction policies.

The simulation algorithm iterates values of $N_c$, for the short run, and $N_c$ and $N$, for the long run. The simulation ends when the stable, interior equilibrium arises. If such an equilibrium does not exist, the analysis reports the resulting corner solution. Simulation requires assumptions on parameter values, which may come from a variety of technical sources, government statistics, and academic research. However, these parameter values, and the functional forms used, are only suggestive and should not be taken to represent actual magnitudes.

All simulations use a reference set of parameters, $\Theta_0$, unless noted otherwise. Free-flow in-vehicle time for a car is half an hour, with $t_0 = 0.5$, corresponding to the typical lower bound of travel time in cities of developing countries (UNCHS, 2002). In order that the volume-to-capacity ratio, which determines congestion, may rise to a high level, the
highest potential traffic volume should be greater than road design capacity. Therefore, the total consumer population is $\tilde{N} = 1,000$ per hour, and practical road capacity is $k_r = 650$ cars per hour. The maximum volume-to-capacity ratio, 1.54, along with the standard parameters values, $\alpha = 0.15$ and $\beta = 4$, yields a maximum travel time of almost an hour, similar to the observed one-way commute time in congested megacities. In determining traffic volume, buses are equivalent to three passenger cars, with $e_b = 3$. Because buses carry more passengers than cars, with $k_b = 20$,\(^{12}\) the passenger car equivalent of each bus user is $e_b/k_b = 0.15$. Bus in-vehicle time is greater than car in-vehicle time, with $t_b = 1.5$ (McKnight et al., 2004).

The income parameters are also illustrative of developing countries. Income scale is initially low, with $\mu = 1$, but increases during each simulation to demonstrate the effect of income growth. The initial hourly income of the median income commuter is $Y(\tilde{N}/2) = 2.72$ dollars. Assuming an 8 hour work day, 5 days a week and 50 weeks a year, annual income would be approximately 5,500 dollars, a plausible median income in a developing country. Inequality is initially low, with $\sigma = 0.6$ giving a Gini index of 33. In developing countries, the observed Gini index ranges from the 30s to the 60s (World Bank, 2007). The cost of car travel, $c_c = 4.5$, is three times greater than bus travel, $c_b = 1.5$, and is a large portion of total income (2 trips each workday would cost 2,250 dollars annually for car travel and 750 for bus travel) as is typical in developing countries (Vasconcellos, 2001). The value of an hour of travel time is half of hourly income, assuming there is no income tax, with $v = 0.5$ (Brownstone and Small, 2005).

Following the value of time literature (Wardman, 2001), bus wait time is valued at twice in-vehicle time ($w = 2$). Trip value, is one and a half times hourly income ($h = 1.5$).

\(^{12}\)Informally operated buses, come in a wide range of types and sizes. The minibus, or jitney, a common informal transport vehicle in many developing countries, is typically smaller than a municipal bus and seats between 12 and 24 passengers, with some crowding (Cervero, 2000).
Consumer surplus, $S$, is the aggregate gain of commuters from travel.\(^{13}\) For a mode split $N_c$, with utility for users of each mode defined in equations (9) and (10), surplus is

$$S = \int_1^{N_c} V_c(i, N_c, N, \Theta)di + \int_{N_c}^N V_b(i, N_c, N, \Theta)di. \quad (33)$$

### 3.1 Effect of rising income scale on mode use

To illustrate the effect of rising income scale in equation (27), the simulation increases $\mu$ from 1 to 5, reporting the short-run and long-run equilibria as well as other diagnostics.

First, to emphasize the effect of the lognormal income distribution’s density on mode choice, consider a model where travel time is constant, with $T_c = 0.5$ and $T_b = 0.75$ (Figure 3). All consumers must commute in the short run, with $N = 1,000$. An income scale $\mu = 1$ results in universal bus use because low consumer incomes and values of time do not justify the money cost premium of car travel. Rising income scale gradually increases car use according to the (lower) “S-curve” in Figure 3. As explained in the comparative statics section, car use without congestion rises moderately at first ($\mu = 1.5$ to 2.5) because the top of the income distribution, with the highest income individuals, has low density. However, when incomes of the high density portion of the income distribution rise to a level that justifies car travel ($\mu = 2.5$ to 3.5), car use expands rapidly. When $\mu = 3.5$ to 4.5, and only those at the bottom of the income distribution (also low density) continue to travel by bus, the adoption of car use in response to rising income scale declines.

Return now to the model where travel time responds to traffic volume as specified in equations (20) and (21). Feedback between traffic volume and travel time savings compounds the adoption of car use and decline in bus use, according to the steeply ascending and descending curves in Figure 3. The first column of Table 1 indicates the

\(^{13}\)For numerical results, $S$ is the summation of consumer utilities.
income scale, $\mu$, by increments, and remaining columns report equilibrium mode use, $N_j$, mode travel time, $T_j$, utility compared to not traveling for the highest, median, and lowest income consumers, and consumer surplus, $S$. As car use rises, travel time for both modes increases, following equations (7) and (8). As the median income commuter switches to car travel by $\mu = 3$, bus travel time has increased by almost 13.5 percent while car travel time has only increased by about 11 percent, the difference due to increased wait time. A subsequent increase in income scale to $\mu = 3.1$ pulls the remainder of the middle class to car travel, increasing congestion and bus wait time to a point where bus travel is unsustainable, even for the low income segment. With universal car use, congestion increases car travel time to nearly an hour, so that those switching from bus travel to car travel face even greater travel time than bus travel at $\mu = 3$. The demise of bus service and jump in congestion coincides with a drop in consumer surplus. Low income commuters have the greatest proportional drop in utility from travel.

Note the differences between the short-run and long-run portions of Table 1. While all consumers must commute in the short run, the lowest income commuters have negative utility at $\mu = 1, 1.5$ and 2. For the long-run equilibrium, where $N$ may fall below $\bar{N}$, these consumers choose not to commute and gain 0 utility. The initial increases in income scale expand both bus use and car use, without increasing travel time, representing the early stage of motorization in developing countries. When travel time jumps with the demise of bus service at $\mu = 3.1$, the lowest income consumers again choose to not commute.

### 3.2 Effect of income inequality on mode use

Next, Figure 4 displays long-run equilibrium car and bus use for increasing levels of inequality, $\sigma = 0.6, 0.8$, and 1.0, along with income scales rising from $\mu = 1$ to $\mu = 4$, as in equation (28). The levels of inequality correspond the Gini indices of 32.6, equivalent
to Bangladesh’s national Gini index; 42.5, equivalent to Thailand’s; and 51.6, equivalent to Peru’s (World Bank, 2007).

When \( \sigma = 1 \), the density of the income distribution is flatter, with more high and low income consumers. For all income scales \( \mu < 3 \), more consumers have a value of time sufficient to justify car use. At the same time, the drop in trip value for the lowest income consumers reduces bus use. At approximately \( \mu = 3 \), where the median income commuter is the marginal car commuter, inequality has no effect on mode use. For income scales \( \mu > 3 \), where the majority of consumers travel by car, higher inequality reduces car use and delays the collapse of bus use. Even at income scale \( \mu = 3.5 \), when bus travel time is 1.32 hours and car travel time is 0.76 hours, low income bus commuters still do not have sufficient value of time to prefer car use.

In summary, as income scale rises, a developing country with high inequality will experience a relatively steady increase in car use, while a country with low inequality will experience an abrupt transition to universal car use. A conclusive empirical analysis of this trend would require controlling for additional factors and is beyond the scope of this paper.

### 3.3 Additional simulations

Several other simulations illustrate the sensitivity of the model to variation in parameters. In Figure 5, lower free-flow travel time, \( t_0 \), encourages bus use when the income scale is low, and preserves bus use up to a higher income scale. In Figure 6, higher population, \( \bar{N} \), results in higher overall levels of car and bus use, simply due to the greater number of consumers. Because road capacity remains fixed, these higher use levels generate more congestion which causes bus use to collapse at a lower income scale (a simulation for capacity, \( k_r \), would be similar, without the vertical shift). In Figure 7, a lower multiple
for bus in-vehicle time, \( t_b \), reduces the travel time savings from car use and delays the collapse of bus use until a higher income scale.

## 4 Policy interventions

The drop in consumer surplus associated with the collapse of bus use suggests that equilibrium mode use may not be socially optimal. A social planner, whose goal is to maximize consumer surplus, would choose a socially optimal allocation, \( N_c^{SP} \) and \( N^{SP} \), of consumers to modes and to non-travel. Note first that the pattern of the equilibrium allocation in equation (33) is optimal, with higher income consumers (indexed 1 to \( N_c \)) commuting by car, middle income consumers (indexed \( N_c \) to \( N \)) commuting by bus, and lower income consumers (indexed \( N \) to \( \bar{N} \)) not commuting.\(^{14}\) With the equilibrium allocation pattern optimal, the planner’s decision is then to choose \( N_c \) and \( N \) optimally.

Holding the total number of commuters constant at the optimal level, the optimal mode split requires choosing \( N_c \) to maximize (33) by setting

\[
\frac{\partial S}{\partial N_c} \bigg|_{N=N^{SP}} = V_c(I(N_c), N_c, N^{SP}) - V_b(I(N_c), N_c, N^{SP})
\]

\[
+ \int_{1}^{N_c} \frac{\partial V_c}{\partial T_c} \frac{\partial T_c}{\partial N_c} \, di + \int_{N_c}^{N} \frac{\partial V_b}{\partial T_b} \frac{\partial T_b}{\partial N_c} \, di = 0. \tag{34}
\]

Using equations (7) and (8), the last two terms in (34) are both negative. Therefore, the sum of the first two terms, representing the utility difference of car and bus travel for the marginal car commuter, must be positive at the social optimum (rather than zero, as at

---

\(^{14}\)To see how this pattern is optimal, consider switching the travel modes of two commuters, so that a high income consumer, \( i_L < N_c \), commutes by bus and a lower income consumer \( N_c < i_H < N \) commutes by car. After the switch, \( S \) will be lower because the consumer with lower value of time is using the faster mode. The change in consumer surplus is \([Y(i_H) - Y(i_L)]v[T_b(N - N_c, N_c) - T_c(N_c, N - N_c)] < 0\). Similarly, switching any commuter with a non-commuter would reduce consumer surplus.
an interior equilibrium). Thus, for a socially optimal mode split, at least some commuters with income lower (higher index) than the marginal car commuter would prefer to switch to car travel, a switch the planner does not allow given that other commuters, both car and bus, are hurt by the transfer. The socially optimal mode split must then have fewer car users than the equilibrium mode split, with \( N_{c}^{SP} < N_{c}^{*} \), indicating that equilibrium car use is inefficiently high.

Holding the number of car users constant at the socially optimal level, the social planner chooses the number of total commuters so that

\[
\frac{\partial S}{\partial N} \bigg|_{N_{c}=N_{c}^{SP}} = V_{b}(I(N), N_{c}^{SP}, N) + \int_{N_{c}}^{N} \frac{\partial V_{c}}{\partial T_{c}} \frac{\partial T_{c}}{\partial N} \, di + \int_{N_{c}}^{N} \frac{\partial V_{b}}{\partial T_{b}} \frac{\partial T_{b}}{\partial N} \, di = 0. \quad (35)
\]

As with mode split, when \( N \) increases, the resulting increase in the number of bus users adds to congestion and raises travel time for users of both modes. However, because an increase in ridership also reduces wait time, especially for low levels of bus use, the net effect on bus travel time is ambiguous. Thus, for an optimal allocation where bus use is high and the congestion effect dominates the wait time effect, \( \partial T_{b}/\partial N > 0 \) holds. With the two integrals negative, some non-commuters would prefer to travel by bus and \( V_{b}(I(N), N_{c}^{SP}, N) > 0 \); therefore, \( N_{c}^{SP} < N^{*} \) holds. In contrast, for parameter values where the socially optimal bus use allocation is low, and the wait time effect dominates the congestion effect so that \( \partial T_{b}/\partial N < 0 \) holds, and the sum of the two integral terms may be positive. At this allocation, some bus commuters would prefer not to travel and \( V_{b}(N, N_{c}^{SP}, N) < 0 \); therefore, \( N_{c}^{SP} > N^{*} \) holds.

The inefficiency results of equations (34) and (35) suggest a potential for policy to improve consumer surplus. As will be shown in simulations below, policies that restrain car use, so that \( N_{c}^{SP} < N_{c}^{*} \) holds, result in higher consumer surplus, especially when
the equilibrium allocation would be universal car use. With car use restrained, bus use will be high and only minor adjustments to bus use will improve consumer surplus. For this reason, the analysis focuses on policies targeting the social cost of car use (based on the analysis surrounding (35)). In particular, this analysis illustrates the effect of a car-use toll and reserved bus lanes, two feasible policies which have been implemented in developed and developing countries.

4.1 Toll on car use

By imposing additional monetary costs on car users, a social planner can encourage bus use and reduce congestion. Small (2004) develops a model to show how road pricing complements the virtuous circle of bus service and presents evidence that cordon pricing in London increased ridership and improved bus service. Singapore, seeking to avoid congestion experienced in other fast growing urban areas, implemented electronic road pricing in 1998 (Willoughby, 2001). Despite a high rate of income growth during this period, car ownership in Singapore has remained far below the levels in countries with similar income levels (about 100 cars per 1,000 people), bus mode share has remained high, and congestion is not severe.\footnote{15}

A car use toll, $\tau_c$, reduces car use by increasing the money cost premium of car travel, giving an aggregate consumer surplus of

$$S = \int_1^{N_c} \{V_c(i, N_c, N) - \tau_c\} di + \int_{N_c}^{N} V_b(i, N_c, N)di + N_c \tau_c, \quad (36)$$

where the last term is the contribution of toll revenue to consumer surplus (assuming the planner may allocate revenue for public benefit without loss).

\footnotetext[15]{Many countries in Asia, including Singapore, have increased vehicle registration fees and import taxes to raise revenue and manage car ownership. However, such fees do not directly limit car use.}
With the reference set of parameters and income scale $\mu = 3.5$, the long-run simulation in Table 2.a illustrates the effect of increasing levels of a car-use toll (by increments) on mode use, mode travel time, and consumer surplus. Initially, without a toll, the income scale is sufficiently high that all 1,000 consumers travel by car, with a high level of congestion and $S = 36,618$. Increasing the toll to $\tau_c = 1.0$ deters one consumer from commuting, slightly reducing congestion, but also reduces utility without accounting for toll revenue. A further increase in the toll to $\tau_c = 2$ increases the money cost premium sufficiently to make bus use competitive. Feedback from improved bus service and reduced congestion enables the mass of consumers with low values of time to commute by bus, leaving only 517 car commuters.

Although toll revenue is lower after passing the threshold, consumer surplus is substantially higher, with or without toll revenue. Subsequent toll increases continue to reduce car use (Table 2.a). The toll maximizing consumer surplus without revenue, $\tau_c = 3.5$, allows for only 279 car users (surplus without revenue is 43,220, surplus with revenue is 44,197). However, with revenue, the socially optimal toll is only $\tau_c = 3$ with 336 car users (surplus with revenue is 44,223). Thus, the socially optimal toll reduces car use enough to lower congestion and make bus service viable, but does not eliminate car use altogether.\[16

4.2 Reserved bus lanes

As an alternative to a car use toll, reserved bus lanes are rising in popularity in Latin America and Asia. Curitiba, Brazil, demonstrated success with express bus lanes in 1974, and has retained high bus mode share even as the city has grown. Presently, cities in developing countries around the world are implementing various designs of reserved

\[16\text{Suppose there were no option to travel by bus, the socially optimal congestion toll would be } \tau_c = 17.5, \text{ resulting in } N_c = 822, \tau_c = 0.69, S - N_c \tau_c = 24,618, \text{ and } S = 39,003. \text{ Thus, a socially optimal car use toll increases consumer surplus more when bus use is an option.}\]
bus lanes including Bus Rapid Transit, which operates much like a light rail system and delivers substantial ridership increases and travel time savings (Levinson et al. 2004).

Reserved bus lanes do not necessarily require additional road construction; rather, a share of the existing roadway, $\lambda_b$ ($0 \leq \lambda_b \leq 1$), is dedicated for bus use, while the remainder, $1 - \lambda_b$, is restricted to cars. Thus, travel time between the two modes is no longer interdependent. With reserved bus lanes, car and bus travel times are

$$T_c(N_c) = t_0 \left(1 + \alpha \left(\frac{N_c}{(1-\lambda_b)k_r}\right)^\beta\right),$$

$$T_b(N - N_c) = t_b t_0 \left(1 + \alpha \left(\frac{e_b(N - N_c)/k_b}{\lambda_b k_r}\right)^\beta\right) \left[1 + \frac{wk_b}{2(N - N_c)}\right].$$

This analysis assumes that there is no cost to impose or maintain separation.

As with the car use toll, a planner must choose the road share to allocate for bus use. With the reference parameters and $\mu = 3.5$, Table 2.b illustrates the effect of different road share allocations on long-run mode use. Allocating $\lambda_b = 0$, no road share for buses, is identical to the equilibrium result of universal car use. Reserving even a small share for buses, $\lambda_b = 1/6$, allows buses to travel with virtually no congestion and attracts many commuters that would otherwise travel by car. With bus service viable and reduced congestion for the remaining 576 car users, consumer surplus rises.

Subsequent increases in the share reserved for bus use have little effect on bus travel time, but gradually increase car travel time. The extreme option of prohibiting car use is an improvement over the equilibrium but not socially optimal. Therefore, the socially optimal road share for buses is approximately $\lambda_b = 1/6$, with surplus $S = 43,436$.

---

\(^{17}\)An additional policy option would be to combine access restrictions and tolls. Berglas et al. (1984) show that if two modes have differential contributions and aversions to congestion, separation is optimal and tolling of either mode may further improve welfare. De Palma et al. (2008) discuss a variety of separating and tolling options for managing truck and car traffic. They find that capacity indivisibilities handicap access restrictions from achieving the gains of continuous tolls and that separation may be a user equilibrium.
However, roads are not perfectly divisible, and few city roads have six lanes in one direction. Allocating 1/3 of the road for bus travel, although more than is socially optimal, still achieves most of the benefit of reserved bus lanes, with surplus, $S = 43,362$.

### 4.3 Policy comparison

A final long-run simulation (Table 2.c) compares the effects of a socially optimal car use toll and a socially optimal reserved bus lane allocation on mode use and consumer surplus, as income scale rises from $\mu = 2$ to $4$. Possible bus allocations are $\lambda_b = 0, 1/3, 2/3, \text{ or } 1$ and tolls are $\tau_c \geq 0$. For income scales $\mu < 3$, congestion is minimal and neither a toll nor reserved bus lanes significantly improve consumer surplus ($\lambda_b = 2/3$ is only barely better than $1/3$). For higher income scales $\mu > 3$, where additional car use may cause congestion, both policies prevent traffic volume from passing a critical threshold and there is no jump to universal car use.

However, a socially optimal toll always limits car use and increases consumer surplus more than a reserved bus lane does (Mohring, 1979; Small, 1983). Although reserved bus lanes improve bus service, they also compress the remaining car traffic and thereby increase car travel time, raising the time cost of travel for high income commuters. Nevertheless, reserved bus lanes achieve most of the gains from a car use toll despite the limitation of roads not being perfectly divisible for socially optimal allocation.

### 5 Conclusion

Without policy intervention to maintain bus service, developing countries with income growth are on course toward increased car use and rising traffic congestion. Although empirical evidence at the country level shows a strong association of rising income and car use, at the urban level, local characteristics, congestion, and policies may also affect
travel decisions. First, income inequality may increase motorization at low income scales, and reduce motorization at higher income scales. Thus, countries with lower inequality may experience abrupt motorization as they approach a critical income scale. Second, motorization may be especially abrupt in urban areas due to positive feedback between congestion and car use, ultimately leading to a collapse of bus use. Third, policy interventions, including car use tolls and reserved bus lanes, sustain bus service, reduce congestion, and increase consumer surplus, even as incomes rise.

Besides these results, other factors may also contribute to motorization, and could be the subject of future theoretical or empirical research. First, population growth, \( \bar{N} \), and commute distance, analogous to \( t_0 \), both increase car use and may also be responsible for rapid motorization. A more complete urban transport model might consider the effect of location choice and land use, which determines population and distance, on car use. Second, developing countries have a wide variety of mode choices not available in this analysis, including motorcycles, taxis, carpooling, and non-motorized transportation. A model extension including non-motorized transport as a third mode choice (not presented here) suggests that it is an important alternative only when the income scale is low and when walking or bicycling travel times are comparable to motorized travel times. A further modeling distinction might be made between city provided bus service and informal buses. Third, preferences for car or bus travel may extend beyond time and money costs; while bus travel has crowding disamenities, car travel confers higher social status. Fourth, a political economy model of motorization, modeling mode users as interest groups, could also have positive feedback in mode use due to the positive relationship between the aggregate use of a mode and mode users’ collective lobbying power. Income growth and changes in inequality may alter the size of interest groups and demand for more road capacity, a car toll, or reserved bus lanes.
6 References

Wichiensin, M., Bell, M., Yang, H., 2007. Impact of congestion charging on the transit market: an inter-modal equilibrium model. Transportation Research Part A 41, 703-713.
Figure 1: Utility of consumer $i$ (indexed by decreasing income, trip value, and value of time) for bus or car commuting compared to not commuting, with equilibrium car use, $N_c^*$, bus use, $N^* - N_c^*$, total consumers, $\bar{N}$, and other parameters, $\Theta$, where trip value is greater than time cost for both modes.
Figure 2: (a) The marginal car commuter cost-difference function gives the utility gain from car travel over bus travel for consumer $i$, where $i = N_c$ (see equation (15)), for three parameter levels. The stable equilibrium for car use, $N_c$, is marked $N_c^*$. (b) The marginal commuter cost-difference function gives the utility gain from bus travel over not commuting for consumer $i$, where $i = N$ (see equation (17)). The stable equilibrium for total use, $N$, is marked $N^*$. Note that the scale of $N_c$ in (a) is from 0 to $N$, while the scale of $N$ in (b) is from $N_c^*$ to $\bar{N}$. 
Figure 3: Effect of rising income scale, $\mu$, on short-run, equilibrium car use, $N^*_c$, and bus use, $N - N^*_c$. With fixed travel time there is no congestion or bus waiting time. With feedback, greater traffic volume increases travel time on both modes.

Figure 4: Effect of increasing income inequality, $\sigma$, on long-run, equilibrium car use, $N^*_c$, and bus use, $N^* - N^*_c$, for an ascending income scale, $\mu$. 

39
Figure 5: Effect of variation in free-flow travel time, $t_0$, on long-run, equilibrium car use, $N_c^*$, and bus use, $N^* - N_c^*$, for an ascending income scale, $\mu$.

Figure 6: Effect of variation in the total number of consumers, $\bar{N}$, on long-run, equilibrium car use, $N_c^*$, and bus use, $N^* - N_c^*$, for an ascending income scale, $\mu$. 
Figure 7: Effect of variation in bus in-vehicle travel time relative to car travel time, $t_b$, on long-run, equilibrium car use, $N^*_c$, and bus use, $N^* - N^*_c$, for an ascending income scale, $\mu$. 
Table 1: Effect of rising income scale, $\mu$, on short and long-run, equilibrium car use, $N_c$, and total use, $N$. Diagnostics include mode travel time in hours, utility for consumer $i$ compared to not commuting, and consumer surplus.
<table>
<thead>
<tr>
<th>Car toll $\tau_c$</th>
<th>Commuters $N_c$</th>
<th>N</th>
<th>Travel time $T_c$</th>
<th>$T_b$</th>
<th>Utility $i = 1$</th>
<th>$i = 500$</th>
<th>$i = 1,000$</th>
<th>$S - N_c \tau_c$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1,000</td>
<td>1,000</td>
<td>0.92</td>
<td>infinite</td>
<td>215.5</td>
<td>30.0</td>
<td>0.9</td>
<td>36,618</td>
<td>36,618</td>
</tr>
<tr>
<td>0.5</td>
<td>1,000</td>
<td>1,000</td>
<td>0.92</td>
<td>infinite</td>
<td>215.0</td>
<td>29.5</td>
<td>0.4</td>
<td>36,118</td>
<td>36,618</td>
</tr>
<tr>
<td>1.0</td>
<td>999</td>
<td>999</td>
<td>0.92</td>
<td>infinite</td>
<td>214.6</td>
<td>29.0</td>
<td>0.0</td>
<td>35,652</td>
<td>36,651</td>
</tr>
<tr>
<td>1.5</td>
<td>999</td>
<td>999</td>
<td>0.92</td>
<td>infinite</td>
<td>214.1</td>
<td>28.5</td>
<td>0.0</td>
<td>35,152</td>
<td>36,651</td>
</tr>
<tr>
<td>2.0</td>
<td>517</td>
<td>1,000</td>
<td>0.55</td>
<td>0.86</td>
<td>252.5</td>
<td>34.1</td>
<td>4.0</td>
<td>42,739</td>
<td>43,773</td>
</tr>
<tr>
<td>2.5</td>
<td>411</td>
<td>1,000</td>
<td>0.53</td>
<td>0.82</td>
<td>254.6</td>
<td>34.7</td>
<td>4.2</td>
<td>43,114</td>
<td>44,142</td>
</tr>
<tr>
<td>3.0</td>
<td>336</td>
<td>1,000</td>
<td>0.52</td>
<td>0.80</td>
<td>255.3</td>
<td>35.0</td>
<td>4.2</td>
<td>43,215</td>
<td>44,223</td>
</tr>
<tr>
<td>3.5</td>
<td>279</td>
<td>1,000</td>
<td>0.51</td>
<td>0.79</td>
<td>255.4</td>
<td>35.2</td>
<td>4.2</td>
<td>43,362</td>
<td>44,197</td>
</tr>
<tr>
<td>4.0</td>
<td>234</td>
<td>1,000</td>
<td>0.51</td>
<td>0.78</td>
<td>255.2</td>
<td>35.3</td>
<td>4.3</td>
<td>43,187</td>
<td>44,123</td>
</tr>
</tbody>
</table>

(b) Bus share $\lambda_b$ | Commuters $N_c$ | N | Travel time $T_c$ | $T_b$ | Utility $i = 1$ | $i = 500$ | $i = 1,000$ | $S - N_c \tau_c$ | $S$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1,000</td>
<td>1,000</td>
<td>0.92</td>
<td>infinite</td>
<td>215.5</td>
<td>30.0</td>
<td>0.9</td>
<td>36,618</td>
<td>36,618</td>
</tr>
<tr>
<td>1/6</td>
<td>576</td>
<td>1,000</td>
<td>0.60</td>
<td>0.80</td>
<td>249.8</td>
<td>35.3</td>
<td>4.2</td>
<td>43,436</td>
<td>43,436</td>
</tr>
<tr>
<td>1/3</td>
<td>474</td>
<td>1,000</td>
<td>0.61</td>
<td>0.78</td>
<td>248.5</td>
<td>35.3</td>
<td>4.3</td>
<td>43,362</td>
<td>43,362</td>
</tr>
<tr>
<td>2/3</td>
<td>257</td>
<td>1,000</td>
<td>0.65</td>
<td>0.77</td>
<td>244.2</td>
<td>35.4</td>
<td>4.3</td>
<td>42,956</td>
<td>42,956</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1,000</td>
<td>infinite</td>
<td>0.77</td>
<td>234.8</td>
<td>35.5</td>
<td>4.3</td>
<td>42,679</td>
<td>42,679</td>
</tr>
</tbody>
</table>

(c) Income $\mu$ | No policy | Socially optimal car toll $N_c$ | Socially optimal bus lane $N_c$ | $S - N_c \tau_c$ | $S$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>32</td>
<td>998</td>
<td>8,379</td>
<td>0.1</td>
<td>28</td>
</tr>
<tr>
<td>2.5</td>
<td>157</td>
<td>1,000</td>
<td>14,900</td>
<td>0.4</td>
<td>111</td>
</tr>
<tr>
<td>3.0</td>
<td>532</td>
<td>1,000</td>
<td>25,284</td>
<td>1.2</td>
<td>240</td>
</tr>
<tr>
<td>3.5</td>
<td>1,000</td>
<td>1,000</td>
<td>36,618</td>
<td>3.1</td>
<td>324</td>
</tr>
<tr>
<td>4.0</td>
<td>1,000</td>
<td>1,000</td>
<td>63,293</td>
<td>6.4</td>
<td>378</td>
</tr>
</tbody>
</table>

Table 2: Long-run equilibrium car use, $N_c$, and bus use, $N$, for (a) ascending levels of a car-use toll, $\tau_c$, where income scale is fixed at $\mu = 3.5$; (b) ascending levels of reserved bus lanes with bus road share, $\lambda_b$, where $\mu = 3.5$; and (c) ascending $\mu$ comparing the effects of a socially optimal car use toll and reserved bus lanes.